Dynamics of Stars and Black Holes in Dense Stellar Systems:

Lecture VI:

DYNAMICS AROUND SUPER-MASSIVE BHs

0. nuclear star clusters (NSCs)
1. dynamics around super-massive BHs (SMBHs)
2. formation and dynamics of stars in the central parsec
3. gravitational waves involving NSCs
0. nuclear star clusters (NSCs)

* more massive than globular clusters (M>10^6 Msun)

* MULTIPLE POPULATION (<1 Gyr up to 13 Gyr)

* in lower-mass spheroids (<10^10 Msun), NSCs are more common than SMBHs, but sometimes CO-EXISTENT with the SMBH (Milky Way)

* in high-mass spheroids (>10^11 Msun) SMBH alone in most cases

* NSCs obey SCALING RELATIONs as SMBHs (Ferrarese et al. 2006)

0. Nuclear star clusters (NSCs)

Carson et al. 2015, AJ, 149, 5

= 2” ~ 20 – 40 pc for this sample

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Carson et al. 2015, AJ, 149, 5
0. nuclear star clusters (NSCs)

The NSC we know better is the one of the Milky Way:

* ~ 8 kpc away (100 times closer than the 2nd next)
* co-existent with SMBH
* massive (>10^7 Msun)
* both old (>1 Gyr) and young (~Myr) population
* extremely interesting to understand interplay between stars, gas and SMBH

What do we observe in the Galactic centre?

- A CROWDED ENVIRONMENT!
  - THE SMBH: $4 \times 10^6 \, M_\odot$
  - IONIZED, ATOMIC and MOLECULAR GAS: $>10^6 \, M_\odot$
  - A 'CUSP' of LATE-TYPE STARS: $>10^6 \, M_\odot$ (most of NSC mass)
  - A CROWDED ENVIRONMENT!
  - hundreds of YOUNG STARS
  - several early-type (O and WR) stars
  - ~30 B stars (named S- stars)

0. nuclear star clusters (NSCs)
0. nuclear star clusters (NSCs)

**STARS:**

270 pc < 1/10 scale length of our Galaxy's disc

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Schoedel et al. 2007, A&A, 469, 125
Schoedel et al. 2014 arXiv:1411.4504
OLD STARS: spherical cusp, very high density
\( \sim 10^6 \) M\( \odot \) pc\(^{-3} \)

solar neighborhood \(<\sim 1\) M\( \odot \) pc\(^{-3} \)

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Schoedel et al. 2007, A&A, 469, 125
Schoedel et al. 2014 arXiv:1411.4504
0. nuclear star clusters (NSCs)

YOUNG STARS: <100 Myr

S-cluster r ~ 0.04 pc

0.5 pc ~ 12 arcsec

0. nuclear star clusters (NSCs)

S-cluster

- "cluster" radius ~ 0.04 pc
- ~ 30 stars
- age ~ 20 – 100 Myr
- eccentricity ~ 0.7-0.9
- allow dynamical measurement of SMBH mass (S02, yellow line)

→ The only SMBH candidate for which we can exclude almost every other nature (thanks to S02's orbit):

$$m_{\text{BH}} = 4.30 \pm 0.20 \pm 0.30 \times 10^6 \text{M}_\odot$$

Gillessen et al. (2009)

Credits: Prof. Andrea Ghez, UCLA
0. nuclear star clusters (NSCs)

**Very young stars (aka clockwise disc)**

- between 0.04 and 0.5 pc
- \(~ 10^4\) Msun
- age \(~ 2 – 8\) Myr
- eccentricity \(~ 0.3 +/- 0.1\)
- top-heavy MF (slope\(~ 1.5\))
- between 0.04 and 0.15 pc
- lie in a disc (clockwise disc)

Figure from Yelda et al. 2014, ApJ, 783, 131

1. dynamics around super-massive black holes (SMBHs)

A lot of dynamics close to a SMBH:
- Newtonian precession
- Relativistic precession
- Two-body relaxation
- Resonant relaxation
- Kozai-Lidov (same as star cluster case but with SMBH)
- Tidal capture or Binary break-up (exchange with SMBH)

GAS physics:
- Disruption of molecular clouds
- Accretion on SMBH
- SMBH feedback
- Star formation
1. dynamics around super-massive black holes (SMBHs)

* NSCs are **COLLISIONAL SYSTEMS**
  **BUT ONLY IF SMBH is not included** (increases local velocity field)

* If there are SMBHs, nuclear star clusters are still **COLLISIONAL**
  **OUT OF SMBH INFLUENCE RADIUS** (inside SMBH dominates gravity)

\[
    r_{BH} = \frac{G m_{BH}}{\sigma^2} = 1.7 \text{ pc} \left( \frac{m_{BH}}{10^6 M_\odot} \right) \left( \frac{50 \text{ km } s^{-1}}{\sigma} \right)^2
\]

→ in off-nuclear star clusters two-body relaxation
  is most important effect by far

in NSCs there are other effects at least as
important as two-body relaxation
1. dynamics around super-massive black holes (SMBHs)

**NEWTONIAN PRECESSION(s)**

A star orbiting the SMBH can be described as in Keplerian motion around the SMBH plus an EXTERNAL POTENTIAL (= the old stellar cusp, the other young stars, the CNR)

The external induces PRECESSION

Precession can affect:

- argument of periapsis
- longitude of asc. node
- inclination
- eccentricity

Depending on the structure of the external potential
1. dynamics around super-massive black holes (SMBHs)

- NEWTONIAN PREC. in SPHERICAL POTENTIAL (e.g. spherical stellar cusp):

Timescale

\[ T_{cusp} = \frac{M_{BH}}{M_{cusp}(a)} \cdot P_{orb} \cdot f(e) \]

Only argument of pericentre

- in AXISYMMETRIC POTENTIAL (e.g. stellar or gas ring)

Timescale

\[ T_K = \frac{M_{BH}}{M_{DISC}} \cdot \frac{R_{DISC}^3}{a^{3/2} \sqrt{G \cdot M_{BH}}} \]

- if \( i \sim 0 \) only longitude of ascending node
- if \( i \gg 0 \) also inclination and eccentricity are affected
1. dynamics around super-massive black holes (SMBHs)

- NEWTONIAN PREC. in SPHERICAL POTENTIAL (e.g. spherical stellar cusp):

\[ T_{cusp} = \frac{M_{BH}}{M_{cusp}(a)} P_{orb} f(e) \]

Timescale

SMBH mass \( M_{BH} \)
orbital period of star \( P_{orb} \)
function of eccentricity \( f(e) \)
cusp mass enclosed within semi-major axis \( a \) of stellar orbit

Only argument of pericentre

- in AXISYMMETRIC POTENTIAL (e.g. stellar or gas ring)

\[ T_K = \frac{M_{BH}}{M_{DISC}} \frac{R_{DISC}^3}{a^{3/2} \sqrt{G M_{BH}}} \]

Timescale

disc size \( R_{DISC} \)
disc mass \( M_{DISC} \)
semi-major axis \( a \) of stellar orbit

- if \( i \approx 0 \) only longitude of ascending node
- if \( i \gg 0 \) also inclination and eccentricity are affected
1. dynamics around super-massive black holes (SMBHs)

**RELATIVISTIC PRECESSION:**

precession of orbits in general relativity

Caused by the SMBH mass, even if there are no external potentials
Three types (Schwarzschild prec. + 2 precession effects that depend on spin)

Schwarzschild precession (lowest order correction to Newton):

\[
T_{RP} = 1.3 \times 10^3 \text{ yr} \left(1 - \text{ecc}^2\right) \left(\frac{r}{0.001 \text{ pc}}\right)^{5/2} \left(\frac{4 \times 10^6 \, M_\odot}{M_{BH}}\right)^{3/2}
\]

- affects only argument of pericentre
- efficient for very small semi-major axis
- more efficient for high eccentricity
- more efficient for large SMBH mass
In the case of Galactic centre:

- Relativistic precession important only if $a \ll 0.1$ pc.
- Spherical cusp important at $< 0.3$ pc.
- Circumnuclear disc important at $> 0.3$ pc.

**IF SPHERICAL POTENTIAL DOMINATES over AXISYMMETRIC ($T_{\text{cusp}} \ll T_K$),**

then only precession of argument of pericentre and of longitude of ascending node are not damped.
1. dynamics around super-massive black holes (SMBHs)

**TWO-BODY RELAXATION:** changes ENERGY

\[
T_{rlx} = 0.34 \frac{\sigma^3(r)}{G^2 m_* \rho_*(r) \ln \Lambda}
\]

**RESONANT RELAXATION:** changes ECCENTRICITY

**NO ENERGY**

\[
T_{RR} = 10^4 \text{ yr} \left( \frac{r}{0.001 \text{ pc}} \right)^{3/2} \sqrt{\frac{M_{BH}}{3 \times 10^6 M_\odot}} \left( \frac{10 M_\odot}{m_*} \right) \sqrt{\frac{10^3}{N_*}}
\]
1. dynamics around super-massive black holes (SMBHs)

**RESONANT RELAXATION**

Stars orbiting between the SMBH and the star S exert TORQUES on S, if they belong to a flattened structure (disc)

Such torques REDUCE ANGULAR MOMENTUM, not energy of the nearly Keplerian orbit of S

→ eccentricity of S orbit increases

Rauch & Tremaine 1996, NewA, 1, 149
1. dynamics around super-massive black holes (SMBHs)

\[ M^* = 10 \text{ Msun} \]

\[ \text{ecc} = 0.1 \]

\[ N^* = 10^3 \]

\[ a = 0.001 \text{ pc} \]

\[ M_\text{BH} (M_\odot) \]

\[ a \text{ (pc)} \]

\[ T_{\text{rlx}}, T_{\text{RR}}, T_{\text{RP}}, T_{\text{orb}} \]

\[ M_* = 10 \text{ Msun} \]

\[ \text{ecc} = 0.1 \]

\[ N_* = 10^3 \]
2. formation and dynamics of stars in the central parsec

What is puzzling about the young stars in the central pc?
They should not be there!

STARS FORM from gravitational collapse of dense cold gas in the cores of MOLECULAR CLOUDS

A molecular cloud is disrupted by the tidal field exerted by the SMBH if its density is lower than the Roche density

\[ n_{RL} \sim 10^7 \text{ cm}^{-3} \left( \frac{m_{BH}}{3 \times 10^6 M_{\odot}} \right) \left( \frac{\text{pc}}{r} \right)^3 \]

Typical molecular cloud core density \(< 10^6 \text{ cm}^{-3}\)

The stars cannot form in 'normal conditions' if the cloud is disrupted (Phinney 1989).
2. formation and dynamics of stars in the central parsec

Scenarios to explain the young stars (and in general the formation of NSCs)

- cluster inspiral
- binary break-up
- accretion disc fragmentation
- molecular cloud disruption
Cluster inspiral:

A star cluster spirals toward the SMBH by DYNAMICAL FRICTION and is disrupted by the SMBH tidal field


STAR CLUSTER SINKS BY DYNAMICAL FRICTION

WHILE DISRUPTED BY SMBH SHEAR

~ 1 pc
Cluster inspiral:
A star cluster spirals toward the SMBH by DYNAMICAL FRICTION and is disrupted by the SMBH tidal field

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STAR CLUSTER SINKS BY DYNAMICAL FRICTION
WHILE DISRUPTED BY SMBH SHEAR

SMBH
Molecular cloud disruption:

A molecular cloud is disrupted by the SMBH, but

(i) the residual angular momentum,
(ii) the shocks that take place in gas streams

might lead to the formation of a DENSE DISC, denser than Roche density

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2. formation and dynamics of stars in the central parsec

Stars can form in a gas disc, born from the disruption of a molecular cloud

INGREDIENTS:
* A turbulent molecular cloud $R \sim 15 \text{ pc}, M \sim 10^5 \text{ M}_\odot$
* a SMBH sink particle
* integration with SPH or AMR
* cooling + Planck & Rosseland opacities

MM et al. 2012
2. formation and dynamics of stars in the central parsec

**Binary break-up:**
Several binaries are captured and disrupted by the SMBH

**Before..**

\[ a_c \sim 0.6 \left( \frac{m_{BH}}{M_{bin}} \right)^{1/3} r_{tid} \]

\[ e_c \sim 1 - \frac{r_{tid}}{a_c} \sim 0.97 \]

**..After**

Hills 1991, Perets et al. 2007, Perets & Gualandris 2010
2. formation and dynamics of stars in the central parsec

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3. gravitational waves involving NSCs

i- mergers of stellar BH binaries in NSCs

ii- mergers of stellar BHs with SMBHs

iii- mergers of SMBH binaries
* Same effects as lecture 2017dynamics5.pdf: exchanges, hardening

* Potential well makes it difficult for BH binaries to be ejected after 3body encounters

  \[ \rightarrow \text{more massive BH binaries in galactic nuclei?} \]

3. gravitational waves involving NSCs

* Same effects as lecture 2017dynamics5.pdf: exchanges, hardening

* Potential well makes it difficult for BH binaries to be ejected after 3body encounters


→ more massive BH binaries in galactic nuclei?

3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

* Extreme mass-ratio inspirals (EMRIs)
3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

* Extreme mass-ratio inspirals (EMRIs)

* Frequency of GWs never in LIGO-Virgo range but in space-borne detectors range (LISA)

3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

* Extreme mass-ratio inspirals (EMRIs)

* Frequency of GWs never in LIGO-Virgo range but in space-borne detectors range (LISA)

* Extremely unlikely event:
  
  depends on interplay between
  - relativistic precession (RP)
  - GW decay timescale (GW)
  - resonant relaxation (RR)

HOW?
WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

* RR tends to increase eccentricity of stellar BH orbit

If RR more efficient than GW decay the stellar BH PLUNGES into Schwarzschild radius BEFORE EMITTING observable GWs

WE HAVE NO EMRI, because no GW
3. gravitational waves involving NSCs

WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

* RP tends to change the pericentre of the BH orbit

If RP more efficient than RR
the fast changes of BH pericentre
nullify the effect of RR torques
and BH orbit remains ~ the same

The shielding effect of RP on RR is called
Schwarzschild barrier (Merrit+ 2011, PhRvD, 84, 4024):
RP keeps orbits stable against plunge
WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

∗ If RR more efficient than RP
   → BH plunges into SMBH without observable GW event

∗ If RP more efficient than RR
   → BH avoids plunge-in but remains on a stable circular orbit: it does not merge

∗ A fine-tuning is needed for GW decay to be more efficient than both RR and RP
   → the BH spiral-in into the SMBH and emits observable GWs

LET US LOOK AT THE TIMESCALES
We must calculate the three timescales:

1. RELATIVISTIC PRECESSION (RP) TIMESCALE

\[ T_{\text{RP}} = 1.3 \times 10^3 \text{yr} (1 - e^2) \left( \frac{a}{10^{-3} \text{pc}} \right)^{5/2} \left( \frac{4 \times 10^6 M_\odot}{m_{\text{BH}}} \right)^{3/2} \]

2. GW DECAY TIMESCALE

\[ T_{\text{GW}} \sim 6 \times 10^{12} \text{yr} (1 - e^2)^{7/2} \left( \frac{a}{1 \text{ mpc}} \right)^4 \left( \frac{3 \times 10^6 M_\odot}{m_{\text{BH}}} \right)^2 \left( \frac{10 M_\odot}{m_{\text{bh}}} \right) \]

3. RESONANT RELAXATION (RR) TIMESCALE

\[ T_{\text{RR}} \sim 10^4 \text{yr} \left( \frac{a}{1 \text{ mpc}} \right)^{3/2} \left( \frac{m_{\text{BH}}}{3 \times 10^6 M_\odot} \right)^{1/2} \left( \frac{10 M_\odot}{m_{\text{bh}}} \right) \left( \frac{10^3}{N_*} \right)^{1/2} \]
3. gravitational waves involving NSCs

WHICH INTERPLAY BETWEEN RP, GW DECAY and RR?

In summary,

(1) If $T_{GW} < \min(T_{RR}, T_{RP})$

GW emission is efficient and leads to an EMRI

(2) If $T_{RR} < \min(T_{RP}, T_{GW})$

RR increases the eccentricity till the BH PLUNGES into the Schwarzschild radius of the SMBH, WITHOUT EMRI

(3) If $T_{RP} < \min(T_{RR}, T_{GW})$

RP prevents RR from plunging the BH into the Schwarzschild radius, but the BH is 'stuck' in its orbit and cannot merge. Another perturbation is needed to produce an EMRI or a plunge!

ii- mergers of stellar BHs with SMBHs
3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

The Schwarzschild barrier according to Merritt+ 2011 calculations
3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier: $t_{RP} < t_{RR}$
3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier: $t_{RP} < t_{RR}$

* Most orbits stall when encounter the barrier

RR is efficient: ecc increases

RP is efficient: stable orbit
3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier: $t_{RP} < t_{RR}$

Capture by SMBH (loss cone)
- Most orbits stall when encounter the barrier
- If orbit hits the capture radius when $ecc \approx 1$ we have a plunge-in (no significant GW emission)

RR is efficient: ecc increases

RP is efficient: stable orbit
3. gravitational waves involving NSCs

ii- mergers of stellar BHs with SMBHs

The Schwarzschild barrier according to Merritt+ 2011 calculations

Schw. barrier: $t_{RP} < t_{RR}$

Capture by SMBH (loss cone)

Line for coalescence: $t_{GW} < t_{RP}$

objects entering this line coalesce in Hubble time and EMIT GWs (EMRIs)

plunge-in

RR is efficient: ecc increases

RP is efficient: stable orbit

coalescence
3. gravitational waves involving NSCs

iii- mergers of SMBH binaries

* SMBHs might form binaries (thanks to dynamical friction) after their host galaxies merged

* a SMBH-SMBH binary shrinks by 3-body encounters (same as stellar binaries) until 3-body encounters are no longer efficicents because

\[
\frac{da}{dt} \propto -a^2
\]

* further physics needed to shrink semi-major axis (e.g. dynamical friction in a gas disc)

* if the SMBH-SMBH enters the GW regime it merges by GW decay
References:

* Merritt 2013, Dynamics and Evolution of Galactic Nuclei, Princeton University Press


* Rauch & Tremaine 1996, NewA, 1, 149

* Merrit+ 2011, PhRvD, 84, 4024